**Math 1007**

**Unit 7**

Construct a square with sides equal to \( r \).

<table>
<thead>
<tr>
<th>Problem 1</th>
<th>1: Extend the segment and draw a circle centered at one of the endpoints of the segment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image1" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>2: Draw two larger congruent circles centered where the first circle intersects the extended line segment.</td>
</tr>
<tr>
<td></td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>3: Draw a line segment through the intersection of the two circles drawn in the last step.</td>
</tr>
<tr>
<td></td>
<td><img src="image3" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>This segment will go through the endpoint of the original line segment, perpendicular to the line segment.</td>
</tr>
<tr>
<td></td>
<td><img src="image4" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>4: Repeat the first three steps for the other endpoint of the original line segment.</td>
</tr>
<tr>
<td></td>
<td><img src="image5" alt="Diagram" /></td>
</tr>
<tr>
<td>Problem 2</td>
<td>Construct a hexagon with sides equal to $r$.</td>
</tr>
<tr>
<td>-----------</td>
<td>------------------------------------------</td>
</tr>
<tr>
<td>1: Draw a circle centered at one of the endpoints with the radius of the circle equal to the length of the segment.</td>
<td></td>
</tr>
</tbody>
</table>
| 2: Draw a circle centered at the other endpoint with the radius of the circle equal to the length of the segment.  
The intersection of the two circles will be two additional vertices of the hexagon. |

5: Draw two circles, each centered at one of the endpoints of the original line segment, with radius the length of the segment $r$. The circles will go through the other endpoint of the original line segment.

6: The intersection of these circles with the two constructed perpendiculars are the other two vertices of the square. Draw a line segment connecting these points.
3: At each of the two new vertices, draw circles again of the same radius.

The intersection of these new circles with the first circle are the next vertices of the hexagon.

4: Repeat step 3 with the two new vertices.

The two new circles should intersect the first circle at the same point.

<table>
<thead>
<tr>
<th>Problem 3</th>
<th>Construct a rhombus with a 15° angle and sides equal to ( r ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Draw congruent circles centered at one of the endpoints of the line segment, and at the vertex of a 15° angle.</td>
<td></td>
</tr>
</tbody>
</table>
2: Draw congruent circles, one centered on the intersection of the first circle and the 15° angle, with radius touching the other side of the angle; the second centered on the intersection of the original line segment and the circle.

The intersection of the two circles is a point on the other side of a 15° angle.

3: Repeat the process to draw a 15° angle with vertex at the other endpoint of the original line segment.

4: Draw two circles, each centered at one of the endpoints of the original line segment, with radius the length of the segment \( r \). The circles will go through the other endpoint of the original line segment.

5: The intersection of these circles with the two constructed 15° angles are the other two vertices of the rhombus. Draw a line segment connecting these points.

<table>
<thead>
<tr>
<th>Problem 4</th>
<th>Construct an octagon with sides equal to ( r ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Extend the segment.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>Step</td>
<td>Description</td>
</tr>
<tr>
<td>------</td>
<td>-------------</td>
</tr>
<tr>
<td>2:</td>
<td>As in problem 3, copy a 45° at one of the endpoints of the segment.</td>
</tr>
<tr>
<td>3:</td>
<td>Draw a circle centered at the endpoint used as the vertex of the 45° angle with the radius of the circle equal to the length of the segment. The intersection of the circle with the leg of the angle will be the next vertex of the octagon.</td>
</tr>
<tr>
<td>4:</td>
<td>Repeat steps 2 and 3 with the segment the was just constructed.</td>
</tr>
<tr>
<td>5:</td>
<td>Repeat steps 2 and 3 four more times, working your way from one vertex to the next.</td>
</tr>
<tr>
<td>Problem 4 (cont.)</td>
<td>6: Connect the final two vertices with a line segment.</td>
</tr>
<tr>
<td>------------------</td>
<td>------------------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>If you want, you can construct another 45° angle and</td>
</tr>
<tr>
<td></td>
<td>draw the circles like you did in the previous step.</td>
</tr>
<tr>
<td></td>
<td>If the construction was performed correctly, this</td>
</tr>
<tr>
<td></td>
<td>should match up with the other endpoint on the</td>
</tr>
<tr>
<td></td>
<td>original section.</td>
</tr>
</tbody>
</table>

| Problem 5       | Construct a rectangle with length twice the width and |
|-----------------| width equal to $r$.                                   |
|                 | Proceed at first as you did in constructing the square:|
|                 | 1: Extend the segment and draw a circle centered at one|
|                 | of the endpoints of the segment.                      |
|                 | 2: Draw two larger congruent circles centered where   |
|                 | the first circle intersects the extended line segment.|
|                 | 3: Draw a line segment through the intersection of the |
|                 | two circles drawn in the last step.                   |
|                 | This segment will go through the endpoint of the      |
|                 | original line segment, perpendicular to the line      |
|                 | segment.                                              |
|                 | 4: Repeat the first three steps for the other endpoint|
|                 | of the original line segment.                         |
5: Draw two circles, each centered at one of the endpoints of the original line segment, with radius the length of the segment $r$. The circles will go through the other endpoint of the original line segment.

<table>
<thead>
<tr>
<th><img src="image1" alt="Diagram" /></th>
</tr>
</thead>
</table>

6: Draw two more circles, each centered at the intersection of the perpendicular segments we constructed. The circles should again have radius the length of the segment $r$. The circles will go through the other endpoint of the original line segment.

<table>
<thead>
<tr>
<th><img src="image2" alt="Diagram" /></th>
</tr>
</thead>
</table>

6: The intersection of these circles with the two constructed perpendiculars are the other two vertices of the rectangle. Draw a line segment connecting these points.

<table>
<thead>
<tr>
<th><img src="image3" alt="Diagram" /></th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 6</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>1: Extend the segment and draw a circle centered at one of the endpoints of the segment.</td>
</tr>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
</tr>
<tr>
<td>2: Draw two larger congruent circles centered where the first circle intersects the extended line segment.</td>
</tr>
<tr>
<td><img src="image2.png" alt="Diagram" /></td>
</tr>
<tr>
<td>3: Draw a line segment through the intersection of the two circles drawn in the last step. This segment will go through the endpoint of the original line segment, perpendicular to the line segment.</td>
</tr>
<tr>
<td><img src="image3.png" alt="Diagram" /></td>
</tr>
<tr>
<td>4: Repeat the first three steps for the other endpoint of the original line segment.</td>
</tr>
<tr>
<td><img src="image4.png" alt="Diagram" /></td>
</tr>
<tr>
<td>5: Draw two circles, each centered at one of the endpoints of the original line segment, with radius the length of the segment $r$. The circles will go through the other endpoint of the original line segment.</td>
</tr>
<tr>
<td><img src="image5.png" alt="Diagram" /></td>
</tr>
<tr>
<td>Problem 6 (cont.)</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
</tbody>
</table>
| 6: The intersection of these circles with the two constructed perpendiculars are the other two vertices of the square. Draw a line segment connecting these points. | ![Diagram](image1.png)  
| 7: Connect opposite vertices to draw the diagonals of the square. | ![Diagram](image2.png)  
| Draw a circle with center at the intersection of the diagonals with all four vertices of the square on the circle. | ![Diagram](image3.png)  

Construct a hexagon with sides equal to $r$.

1: Draw a circle centered at one of the endpoints with the radius of the circle equal to the length of the segment.

2: Draw a circle centered at the other endpoint with the radius of the circle equal to the length of the segment.

The intersection of the two circles will be two additional vertices of the hexagon.

3: At each of the two new vertices, draw circles again of the same radius.

The intersection of these new circles with the first circle are the next vertices of the hexagon.

4: Repeat step 3 with the two new vertices.

The two new circles should intersect the first circle at the same point.
<table>
<thead>
<tr>
<th>Problem 7 (cont.)</th>
<th>Connect opposite vertices.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>![Hexagon Diagram]</td>
</tr>
<tr>
<td>What figures are formed inside the hexagon?</td>
<td>There are six equilateral triangles formed inside the hexagon.</td>
</tr>
<tr>
<td>Does this construction suggest another way to construct a hexagon?</td>
<td>Answers will vary. One possibility is:</td>
</tr>
<tr>
<td></td>
<td>Yes, starting with a line segment, construct an equilateral triangle.</td>
</tr>
<tr>
<td></td>
<td>Then extend the two constructed legs of the triangle and construct a second equilateral triangle, upside down, on top of the first.</td>
</tr>
<tr>
<td></td>
<td>Then draw the circle with center the top vertex of the constructed triangle, passing through the other vertices of the triangles we have constructed.</td>
</tr>
<tr>
<td></td>
<td>Then construct the line parallel to the base of the first triangle through the top vertex of the constructed triangle.</td>
</tr>
<tr>
<td></td>
<td>The intersection of the line with the circle will be the other two vertices of the hexagon.</td>
</tr>
</tbody>
</table>